ZARGON ANKAN

Quiz 4, Date: 22 Decembere, 2024

Instructor: Ankan Kar

Timing: 6:00 PM to 8:00 PM — Score rule as per IMO

All questions carry equal marks, try as much as possible
This will be a short quiz

1. Let a and b be positive integers such that $a^2 + b^2$ is divisible by $ab + 1$. Prove that $\frac{a}{b} + \frac{b}{a}$ is an integer.

2. Find all pairs of positive integers (x, y) such that $x^2 + y^2 + 1$ is divisible by xy.

3. Let a, b, c, d be positive integers such that $a^2 + b^2 + c^2 = d^2$. Find the largest positive integer m that divides $abcd$ for all such quadruples (a, b, c, d).

4. Find all 4-tuples (a, b, c, d) of natural numbers with $a \leq b \leq c$ such that

$$a! + b! + c! = 3^d.$$

End